<u>Human Decision Making Under Uncertainty And The Need To Dissociate Mean-Variance Analysis And Expected Utility Theory</u>

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Several papers have examined the conditions under which the behavioral predictions of Mean-Variance (MV) and Expected Utility (EU) models coincide; focusing on the shape of the utility function or subtle measures of risk such as semi-variance. There is, however, a fundamental difference in the two approaches: EU requires the explicit learning of probabilities using, for instance, Bayesian updating, with the drawback that, as the number of state probabilities increases, accurate estimation of probabilities becomes impossible. This learning limitation does not hold for the MV optimizer, who can simply learn the mean and variance using a reinforcement learning algorithm. We conjecture that this fundamental difference has important behavioral effects which we have explored with a novel experimental paradigm.

In this paradigm, the outcome of each gamble is based on random drawing of a ball from a bin. Balls within each bin are distinguished by color. There may be many balls of the same color, but participants do not know how many. Like in standard lotteries, each ball is labeled with a number. This number is the same for balls of the same color and determines how much participants earn when a ball is drawn. For instance, if the red balls are labeled "5", he/she make 5 francs every time a red ball is drawn. Whenever we change the bin, we give the opportunity to try it out, before making decision whether to buy into the gamble or not for a posted price. If they decide to play, the outcome minus the price (net payoff) is added to their play money. Participants can sample as much as they like. The crucial feature of the task is that occasionally, we may change the labels without changing the composition of the bin (number of balls of each color).

States (as determined by the number of colors) and payoff variance are changed independently from one bin to the other. The number of states was fixed at 2, 3, 5, or 10. The payoff standard deviation was fixed at 4, 8, or 12. We hypothesized that as the number of states increases and because this increases the number of probabilities to estimate, an EU maximizer will need to sample more in order the learn the value of the bin. This is not the case for a MV optimizer, who will conversely sample more if the payoff variance increases, in order to get an accurate estimate of the mean. In addition, changes in labels require the MV optimizer to resample the bin because he/she did not keep track of the probabilities, as opposed to the EU decision-maker.

We recruited twenty-seven subjects (10 women, 17 men). At the end of the experiment, subjects received 1/10 of their net play money in real currency.

The first main result is that subjects sampled with high probability (greater than 50%) when we only changed the labels (payoffs) of the balls in the bin, keeping the composition of the bin the same. This is strong evidence that subjects – at least in part – relied on MV analysis.

Next, we analyzed sampling duration (length was set equal to 0 if no sampling). Participants reduced the length of sampling after (only) payoffs changed but increased it when the number of states increased, consistent with the EU approach. There was a significant interaction effect of the payoff change (only) and the number of states: the difference between the length of sampling when a new bin was presented against when only payoffs changed decreased with the number of states. This suggested an increased reliance on MV analysis. In addition and consistent with MV computation, the length of sampling became sensitive to standard deviation when state complexity increased.

Altogether, the results support the notion that subjects mix the EU approach (they increase sampling as state complexity increases) and the MV approach (they re-sample). But when state complexity increases, MV analysis gains weight. Indeed, they re-sample more and the re-sampling length becomes sensitive to payoff variance.

Turning to purchase decisions, we analyzed reaction times (times needed to decide whether to buy a gamble for a certain price). These were expected to be shorter when subjects relied more on MV analysis. Conversely, we expected reaction times to increase with state complexity to the extent that subjects relied on EU analysis. We know that subjects did rely on EU, although less so when the number of states was high. As such, we expected reaction times to increase with the number of states, but at a decreasing rate, whereby reaction times may eventually decrease for high number of states. Results were consistent with this prediction: reaction time increased significantly from 2, 3, and 5 states, but remained constant between 5 and 10 states.

Interestingly, the biggest shift from EU to MV ``thinking'' occurred when increasing the number of states from 5 to 10. We conjecture that this is related to the effective limit to working memory in the human brain, which is said to be able to hold about seven elements. Below seven elements (e.g., when 5 state probabilities need to be accounted for), EU is perfectly manageable. Beyond seven, however, other protocol needs to be followed. We conjecture that humans appeal to mean-variance analysis to overcome inherent limitations in their working memory.

For future research and in order to further support the hypothesis that humans rely more on MV and hence reinforcement learning when the number of states is high, it will be necessary to look at brain activation when participants are sampling from a lottery with low versus high number of states. For low number of states, we expect to observe BOLD signals that covary with probability updating (derived from Bayes' law and Dirichlet priors). For high number of states, we expect to observe BOLD signals that covary with reward and risk prediction errors (derived from a reinforcement learning algorithm).